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This paper deals with the question of the transfer of the diagnostic assessment Pépite at different grade levels. First, we present the theoretical foundations and the modalities of the diagnostic assessment Pépite. Then, we characterize the model of the diagnostic test and we study how this model is compatible according to different grade levels. We detail the response analysis and also we explain how to transfer it to different grade levels.

Keywords: Diagnostic assessment, Information and Communication Technology (ICT), elementary algebra, student's profile, teaching suggestions.

CONTEXT

Teachers are looking for tools in order to help their students. In fact, to allow each student academic progress, teachers need detailed diagnosis about individual student's learning. However, teachers also need to manage the whole class by proposing differentiated activities which are adapted to groups of learners with close competences or who require the same teaching strategy.

This paper addresses the TWG15 "Teaching mathematics with resources and technology". Our research concerns the development and the use of online resources for diagnosis and differentiated learning in the field of elementary algebra. It takes place into the *Pépite* project whose objective is to design and implement a web-based application to support mathematics teachers in managing the cognitive diversity of their students in school algebra classes (Delozanne, Prévit, Grugeon-Allys, & Chenevotot-Quentin, 2010).

Since 2011, we spread our research tools on *LaboMep* (Pilet, Chenevotot, Grugeon, El-Kechaï, & Delozanne, 2013), the online databank developed by *Sésamath*, a French mathematics teachers association. The success of the *LaboMep* platform shows us that such online resources may answer to the teachers' needs (Artigue & Gueudet, 2008): they are looking for valuable information about consistency and misconceptions in student's algebraic activity. First, we implemented the diagnostic assessment *Pépite* for $9^{th}/10^{th}$ grade students (15–16 years old). Second, we implemented a tool providing automatically teaching suggestions according to the learning objectives aimed at by the teacher and adapted to students' diagnostic assessment (Pilet et al., 2013).

This paper deals with the question of the transfer of the diagnostic assessment *Pépite* at different grade levels. First, we present the modalities of the diagnostic assessment *Pépite* and the theoretical elements mobilized. Then, we characterize the model of the diagnostic test and we study how this model is compatible according to different grade levels. We detail the response analysis and we also explain how to transfer it to different grade levels. Last, we conclude with some research perspectives.

THE DIAGNOSTIC ASSESSMENT PEPITE

The theoretical foundations

The modeling presented here is not based on a psychometric approach. The *Pépite* diagnostic assessment is based firstly on an epistemological and anthropological approach and secondly on a cognitive approach of elementary algebra in order to define a reference (Artigue, Grugeon, Assude, & Lenfant, 2001).

In its *tool* dimension (Douady, 1986), the algebra field covers traditional arithmetic problems, problems of generalisation and proof, problems where algebra appears as a modelling tool, problems to put into equation, algebraic and functional problems (Chevallard, 1989). In its *object* dimension, algebra is a structured set of objects with specific properties, semiotic representations and treatment modes taking into account both their semantics and their syntax (Kieran, 2007; Vergnaud, Cortes, & Favre-Artigue, 1988). The diagnosis *Pépite* relies on a multidimensional analysis of the algebraic activity (Grugeon, 1997; Kieran, 2007 [1]) which allows identifying consistency in student's algebraic activity and following its evolution.

According to the anthropological approach, mathematical knowledge strongly depends on the institutions where it has to live, where it has to be learnt or to be taught. Mathematical objects do not exist per se but emerge from practices, which are different from one institution to another one. Chevallard (1999) analyses them in terms of praxeology, i.e., in terms of type of tasks, techniques used to solve these tasks (praxis), technological discourse developed to explain and justify particular techniques, and last, theories which structure the discourse (logos). Here, the diagnostic assessment *Pépite* depends on the curriculum at the end of compulsory education. At each grade level, diagnostic tasks are characterized by a type of tasks, the complexity of algebraic objects involved and the level of involvement of tasks in the resolution.

Pépite diagnostic tasks

The *Pépite* test is composed of ten diagnostic tasks that cover the algebraic field grouped into four sets of types of tasks: calculus (developing or factoring algebraic expressions, solving equations), production (of expressions, formulas or equations), translation or recognition of mathematical relationships from one register of representation to another, solving problems in different mathematical frameworks (numeric, algebraic, geometric, functional) with algebra in order to generalize, prove properties, model or put into equation. The diagnostic tasks may be multiple-choice questions or open responses with multistep reasoning.

The conceptual IT model of classes of tasks developed by Prévit (Delozanne, Prévit, Grugeon, & Chenevotot, 2008) allows characterizing equivalent tasks on a diagnosis point of view. Prévit developed *PépiGen*, software that automatically generates the tasks and their analysis. It uses *Pépinière* software that generates anticipated student's correct or incorrect responses, according to a priori analysis of the tasks.

We work on two main points to transfer *Pépite* assessment at several grade levels. On the didactic modelling side, we have to define a set of tasks that cover the mathematical field at the grade level considered and the associated didactical variables. On the IT modelling side, we have to build generic tests in order to have a same framework for each test.

The response analysis

The diagnostic assessment *Pépite* includes three stages:

- The *local diagnosis* (on a single exercise) analyses each student's response on several dimensions and not only in terms of correct/incorrect and the diagnostic system provides a set of codes that characterize this response according to types of anticipated responses;
- The *individual global diagnosis* (on a set of exercises) collects similar codes on different exercises to build the student's cognitive profile expressed by a level at three scales of skills, success rates and personal features (relative strengths and limitations, false rules and correct rules);
- The *collective global diagnosis* defines groups of students who have close cognitive profiles.

In order to transfer the response analysis, for every task, we need to anticipate the different types of responses and their different forms and study if the computer algebra system *Pépinière* allows automated analysis. At last, we have to build the algorithm to calculate the student's profile.

We now expose the issue of the transfer of the diagnostic assessment *Pépite* at the $7^{th}/8^{th}$ grade level and define some conditions to succeed.

THE TRANSFER OF THE DIAGNOSTIC TASKS

The first step of the transfer of *Pépite* concerns the design of diagnostic tasks with two necessary conditions: theoretical foundations and institutional constraints.

Types of tasks	Number of items	Test item
Calculus	4 / 27	5.1 / 5.2 / 5.3 / 5.4
Producing algebraic expressions	6 / 27	3.1 / 8.1 / 8.2 / 8.3 / 9 / 10.2
Translation or recognition	16 / 27	1.1 / 1.2 / 1.3 / 1.4 / 2.1 / 2.2 / 2.3 / 3.2 / 4.1 / 4.2 / 4.3 / 4.4 / 4.5 / 6 / 7 / 10.1
Problem solving in different mathematics frameworks	3 / 27	8.3 / 9 / 10.3

Table 1: Organization of the 9th/10th grade level test in terms of types of tasks

Types of tasks	Number of items	Test item
Calculus	4 / 22	7.1 / 7.2 / 8.1 / 8.2
Producing numerical expressions	1 / 22	5
Producing algebraic expressions	3 / 22	3.1 / 5 / 6
Translation or recognition	14 / 22	1.1 / 1.2 / 1.3 / 2.1 / 2.2 / 2.3 / 3.2 / 4.1 / 4.2 / 9.1 / 9.2 / 9.3 / 9.4 / 10
Problem solving in different mathematics frameworks	1 / 22	6

Table 2: Organization of the 7th/8th grade level test in terms of types of tasks

A tasks transfer that ensures the mathematical coverage area

To ensure that the test takes into account all types of tasks involved in the algebraic field, we characterize each item of a diagnostic task by one or more of these types of tasks (Chenevotot-Quentin, Grugeon, & Delozanne, 2011). We consider that the diagnostic test covers the types of tasks in the field if all types of tasks are involved. As shown in Table 1, the ten diagnostic tasks of the initial test (9th/10th grade level) cover the algebraic field. The test for the 7th/8th grade level is also composed of ten diagnostic tasks. The Table 2 shows that they are spread across all types of tasks. As at this grade level, numeric skills play an important role in the entry into the algebraic activity, we add a new type of tasks to our list: *Produce a numerical expression*. We assume that the question of whether a student may or may not produce a numerical online expression with correct parentheses is an important indicator of its interpretation of algebraic expressions. We illustrate this task in the following text.

A transfer by adapting existing tasks or adding new tasks

To comply with the curriculum experienced by students in grade 7th in France, the transfer of the test

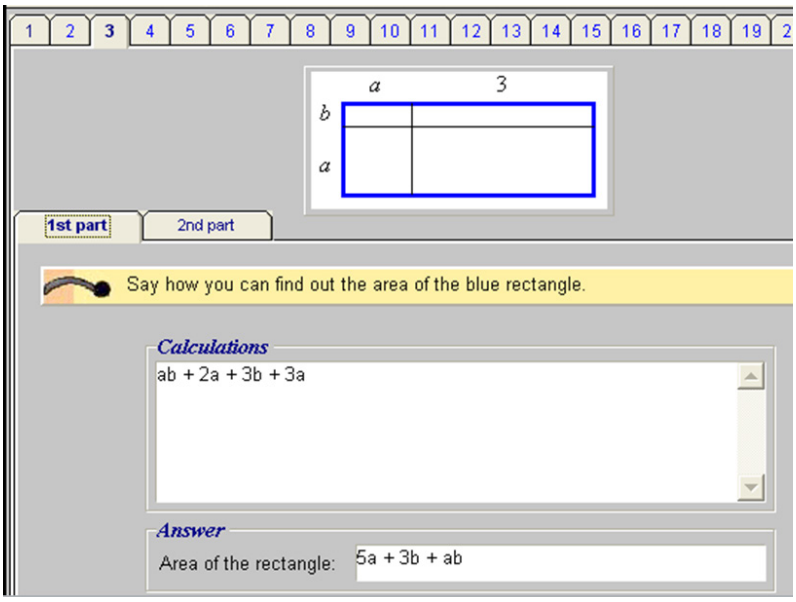
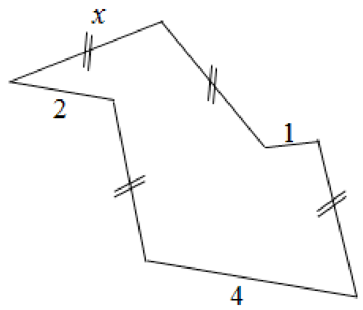


Figure 1: Item 3.1 of the initial test



Shows how to calculate the perimeter of the figure	
Draft for calculations	Perimeter of the figure

Figure 2: Item 3.1 of the 7th/8th grade level test: an example of an adapting task

13 girls and 15 boys go to the movies. Each pays his ticket € 6.80, buys one soda 3 €, popcorn € 3.20 and also one glass € 2.50. Write one line expression to find the amount spent by the group without doing calculation.
Expression of the amount spent by the group

Figure 3: Item 5 of the 7th/8th grade level test: an example of a new task

requires a specific work on tasks. We distinguish, on the one hand, characteristic tasks of the algebraic field that are transferred with adaptations from the 9th/10th grade to a lower grade, and, on the other hand, tasks of the numerical field, specifically designed for 7th/8th grade level. We adapt the tasks of the algebraic field by adjusting the values of didactical variables such as the structure of algebraic expressions or the choice of numbers. These adaptations are justified by both the curriculum and the algebraic activity expected at this grade level. We present an example of adapting item: adapting the item 3.1 of the initial test (Figure 1) for the 7th/8th grade level test (Figure 2). In both cases, the type of tasks is *Produce an algebraic expression in the geometric setting*. In the initial (resp. new) test, the task relates to the area (resp. perimeter) of a rectangle (resp. figure). The area of the rectangle is a second-degree expression and the structure is too complex for the 7th/8th level. That is why we have chosen for this grade level a figure leading to a first-degree expression. Moreover, this choice allows identifying students that concatenate all terms (11x response) or those who are still in a repeated addition ($x+x+x+x+7$ response).

To take into account the numerical skills of students who just discover algebra, we design new tasks. Figure 3 shows one of these tasks which belongs to the type of task: *Produce a numerical expression*. This task assesses if students can correctly produce one numerical expression with parentheses or if "step by step" reasoning persists.

THE TRANSFER OF THE RESPONSE ANALYSIS

We expose the issue of the transfer of the response analysis in the diagnostic assessment *Pépite* by successively visiting the three stages of the process: local diagnosis, individual diagnosis and collective global diagnosis.

First stage: Local diagnosis

The whole process of the diagnostic assessment *Pépite* relies on the quality of this first step: assessment of each student's responses. Students' responses are not only evaluated in terms of correct/incorrect. They are also coded in terms of consistency in the student's algebraic activity, determined by *a priori* analysis (skills, recurring errors). We define six analysis codes found-

Tick the correct equation			
$\frac{1}{2}+\frac{1}{3}=\frac{3}{2}$	$\frac{1}{2}+\frac{1}{3}=\frac{2}{5}$	$\frac{1}{2}+\frac{1}{3}=\frac{2}{6}$	$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

Choice	A priori analysis	Code
1	Incorrect calculation based on the cross product	V3 EA5
2	Addition of numerators and of denominators	V3 EA42
3	Addition of numerators and product of denominators	V3 EA33
4	Correct	V1 EA1

Figure 4: Local diagnosis of the item 1.4 for the 9th/10th grade level test

Tick the correct equation			
$\frac{2}{3}+\frac{1}{6}=\frac{3}{6}$	$\frac{2}{3}+\frac{1}{6}=\frac{3}{9}$	$\frac{2}{3}+\frac{1}{6}=\frac{3}{18}$	$\frac{2}{3}+\frac{1}{6}=\frac{5}{6}$

Choice	A priori analysis	Code
1	Addition of numerators without putting the same denominator	V3 EN33
2	Addition of numerators and of denominators	V3 EN42
3	Addition of numerators and product of denominators	V3 EN33
4	Correct	V1 EN1

Figure 5: Local diagnosis of the item 1.1 for the 7th/8th grade level test

ed on theoretical study: validity of the response (V), meaning of the equal sign (E), use of letters as variables (L), algebraic writings produced during symbolic transformations (EA), representations used during translating a problem (T) and level of justification (J). Figure 4 and 5 show the responses of a 9th/10th and 7th/8th grades student to the task exposed in Figure 1 and 2 and their analysis.

Are these six analysis codes adequate to transfer the assessment to 7th/8th grade student? It is necessary to complete the six previous codes by adding two new codes to study the numerical writings produced during symbolic transformations (EN) and the skills with negative and decimal numbers (N). The EN code is created to take into account that the algebraic skills are built from numerical skills. As the EA and the EN codes have a close structure, we do not distinguish EA and EN in the initial test.

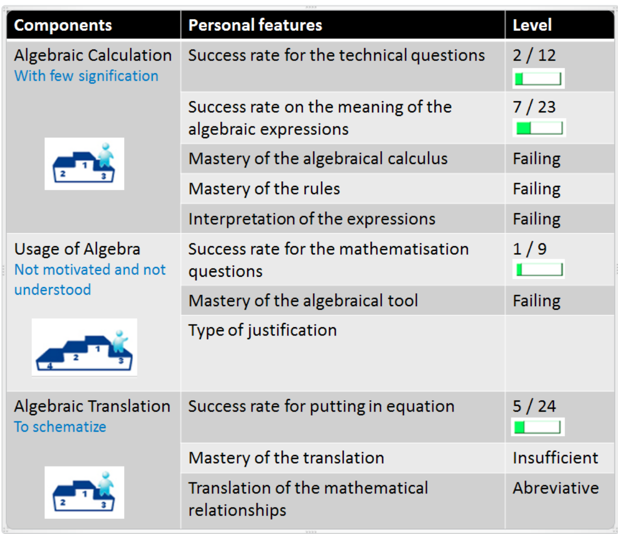
The local diagnosis is based on the anticipated responses obtained both by a didactical analysis and a corpus of responses; it produces the analysis grid. This method is adapted to the IT designing and computer programming. The two pieces of software *PépiGen* and *Pépinière* automatically generate tasks, their analysis and anticipated students' responses. The computer programming of the multiple-choice questions is easier than the one line open responses. In this case, the analysis is effective and generic. Around 10 to 15% of the open responses are still not analysed because of the complexity of the algebraic reasoning, sometimes written with a French text. The analysis of open responses needs specific treatments.

Second stage: Individual global diagnosis

The individual global diagnosis operates on the set of the ten exercises. The system analyses student's responses and calculates the student's cognitive profile through a transversal analysis of the codes of all their responses.

To build the student's cognitive profile, we define a scale of skills with three components founded on theoretical study: Use of Algebra for solving problems (coded UA); flexibility in translating different types of representations (geometric figures, graphical representations, natural language) into algebraic expressions and vice versa (coded TA); ability and adaptability in various uses of algebraic calculation (coded CA). For each of the three components, we identify a scale with different modes, and appropriate criteria for each (Delozanne, Vincent, Grugeon, Gélis, Rogalski, & Coulange, 2005). Figure 6 shows the individual global diagnosis for a 9th grade student with CA3-UA3-TA3 (Figure 6). This student does not give much sense to algebraic activity and does not use it as a tool for solving problems.

To transfer assessment to other grade levels, we also need to complete the algorithm to compute the profile by adding a fourth component to evaluate the various



Components	Personal features	Level
Algebraic Calculation With few signification	Success rate for the technical questions	2 / 12
	Success rate on the meaning of the algebraic expressions	7 / 23
	Mastery of the algebraical calculus	Failing
	Mastery of the rules	Failing
	Interpretation of the expressions	Failing
Usage of Algebra Not motivated and not understood	Success rate for the mathematisation questions	1 / 9
	Mastery of the algebraical tool	Failing
	Type of justification	
Algebraic Translation To schematize	Success rate for putting in equation	5 / 24
	Mastery of the translation	Insufficient
	Translation of the mathematical relationships	Abreviative

Figure 6: 9th grade student's cognitive profile

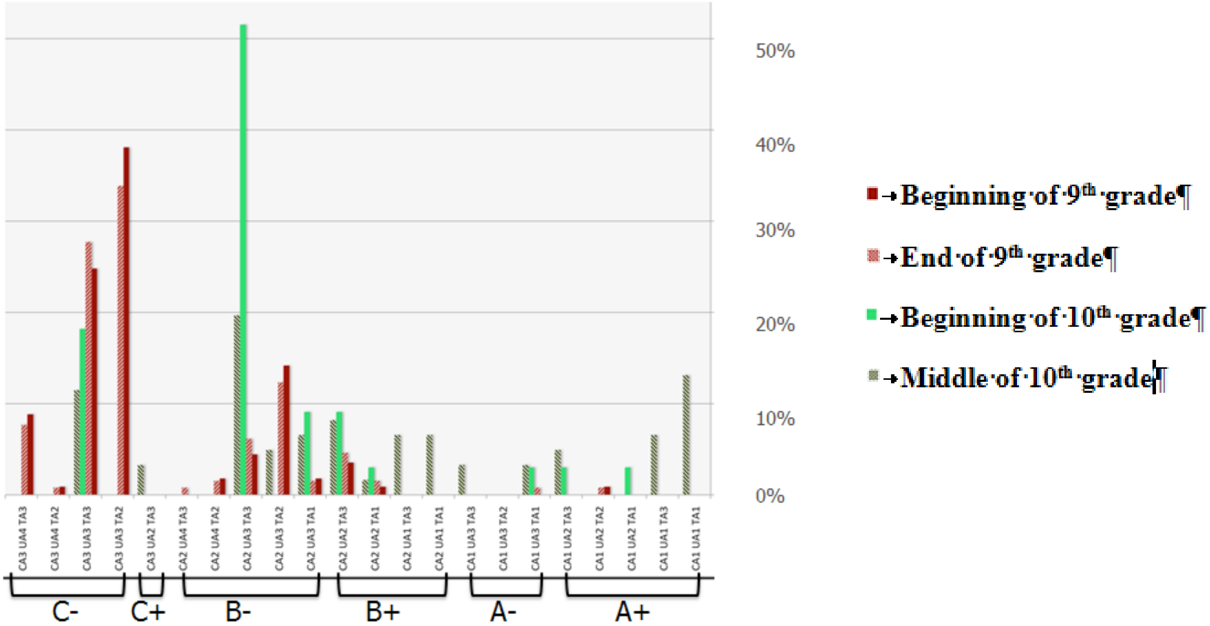


Figure 7: Collective global diagnosis for $9^{th}/10^{th}$ grade students (191 persons)

uses of numerical calculations (coded CN) and modes on the literacy scale. The CN component is created to take into account that the algebraic skills are built from numerical skills. This is the reason why the computation of the modes on scale on the CA and CN components has a similar algorithm.

Third stage: Collective global diagnosis

Teachers request that the diagnosis allows defining groups of students according to their skills in algebra with the aim of setting up strategies of differentiation. *PépiMep* software automatically calculates three groups of students (groups A, B and C) who have close profiles in algebra (Grugeon-Allys, Pilet, Chenevotot-Quentin, & Delozanne, 2012). For $7^{th}/8^{th}$ grade, the algorithm to calculate groups takes into account the new CN component.

CONCLUSION

Different studies comparing diagnostic assessment *Pépite* to other forms of assessment show that *Pépite* is reliable and valid, even with open questions (Delozanne et al., 2008, Delozanne et al., 2010). Overall, it provides a tool to save time and to avoid a very tedious work of a human being.

Since 2011, we implemented in *LaboMep* one test for 9^{th} grade students (14–15 years old) and two tests for $9^{th}/10^{th}$ grade students (15–16 years old). We realized the model of the diagnostic tasks and we designed test and *a priori* analysis for $8^{th}/9^{th}$ grade students and $7^{th}/8^{th}$ grade students. We are now programming them in *LaboMep*.

From the theoretical foundations, we define a scale related to the algebraic activity. We can thus follow the evolution of the algebraic skills of the students on different grades. 191 students passed the 9^{th} test and the $9^{th}/10^{th}$ test and we have already observed that the skills of the students increased. We project to validate this result from the 7^{th} grade to the 10^{th} grade (Figure 7).

What are the research perspectives? We project to extend the tool providing automatically teaching suggestions according to the learning objectives aimed at by the teacher and adapted to students' diagnostic assessment (Pilet et al., 2013), from 7^{th} to 10^{th} grade.

Do the following three calculation programs give the same result?		
Program 1	Program 2	Program 3
Choose a number, Multiply that number by 4, Add 3 to the product obtained.	Choose a number, Multiply that number by 7.	Choose a number, Multiply that number by 4, Add the product obtained triple the number selected.

Figure 8: Learning situation about equivalence of expressions

The first step, transfer of the diagnostic assessment to 7th/8th grade, is already realized. The second step will be to adapt learning situations already defined to study some epistemological algebraic aspects, often not sufficiently explained in curricula: sense of algebraic tool for solving problems of generalization, equivalence of expressions (Grugeon-Allys et al., 2012). The example given in Figure 8 illustrates a situation adapted to the target goal. This research is part of the NéOPRAEVAL project from the French *Agence Nationale de la Recherche*.

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ENDNOTE

1. Using a synthesis of international researches in didactics of algebra, Kieran (2007) proposed the GTG model which differentiates three complementary algebraic activities: (1) Generative activities involve producing various algebraic objects (expressions, formulas, equations and identities), (2) Transformational activities involve the usage of transformational rules (factorization, expansion of products, rules for solving equations and inequalities…), (3) Global/meta-level activities involve the mobilization and use of the algebraic tool to solve different types of problems (modeling, generalization, proof).